Astrophysics with Gravitational Waves

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Introduction

- A brief review of physics and astrophysics with gravitational waves.

- Contents:
  - Gravitational Waves and their Generation
    based on the quadrupole formula
  - Sources of Gravitational Waves
  - Physics and Astrophysics with Gravitational Waves
    focusing on GWs from compact binary coalescence

Gravitational Waves

- I will write **a small perturbation** $h_{\mu\nu}$ on a flat background spacetime $\eta_{\mu\nu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \left|h_{\mu\nu}\right| \ll 1$$

- There are some degree of the freedom in choice of gauge. It is known that **the transvers-traceless (TT) gauge** defined by

$$h^{\mathrm{TT}}_{0\mu} = 0, \quad \eta^{ij}\partial_i h^{\mathrm{TT}}_{jk} = 0, \quad \eta^{ij} h^{\mathrm{TT}}_{ij} = 0,$$

is convenient for discussion of GWs.

- In this gauge, the Einstein equation outside the source, where $T_{\mu\nu} = 0$, is reduced to **a wave equation**

$$\eta^{\mu\nu}\partial_\mu\partial_\nu h^{\mathrm{TT}}_{ij} = \left(-\frac{\partial^2}{c^2\,\partial t^2} + \Delta\right) h^{\mathrm{TT}}_{ij} = 0$$

Gravitational Waves

- The wave equation $\eta^{\mu\nu}\partial_\mu\partial_\nu h_{ij}^{\mathrm{TT}} = \left(-\frac{\partial^2}{c^2\,\partial t^2} + \Delta\right) h_{ij}^{\mathrm{TT}} = 0$

  has **a plane wave solution:**

$$h_{ij}^{\mathrm{TT}}(t,z) = A_{ij}^{\mathrm{TT}} \cos\left(\omega(t - z/c)\right)$$

Here I set the direction of wave propagation as z-axis.

Polarization

- From the TT condition, $h_{0\mu}^{\mathrm{TT}} = 0, \quad \eta^{ij} h_{ij}^{\mathrm{TT}} = 0, \quad \eta^{ij} \partial_i h_{jk}^{\mathrm{TT}} = 0,$ non-zero components are

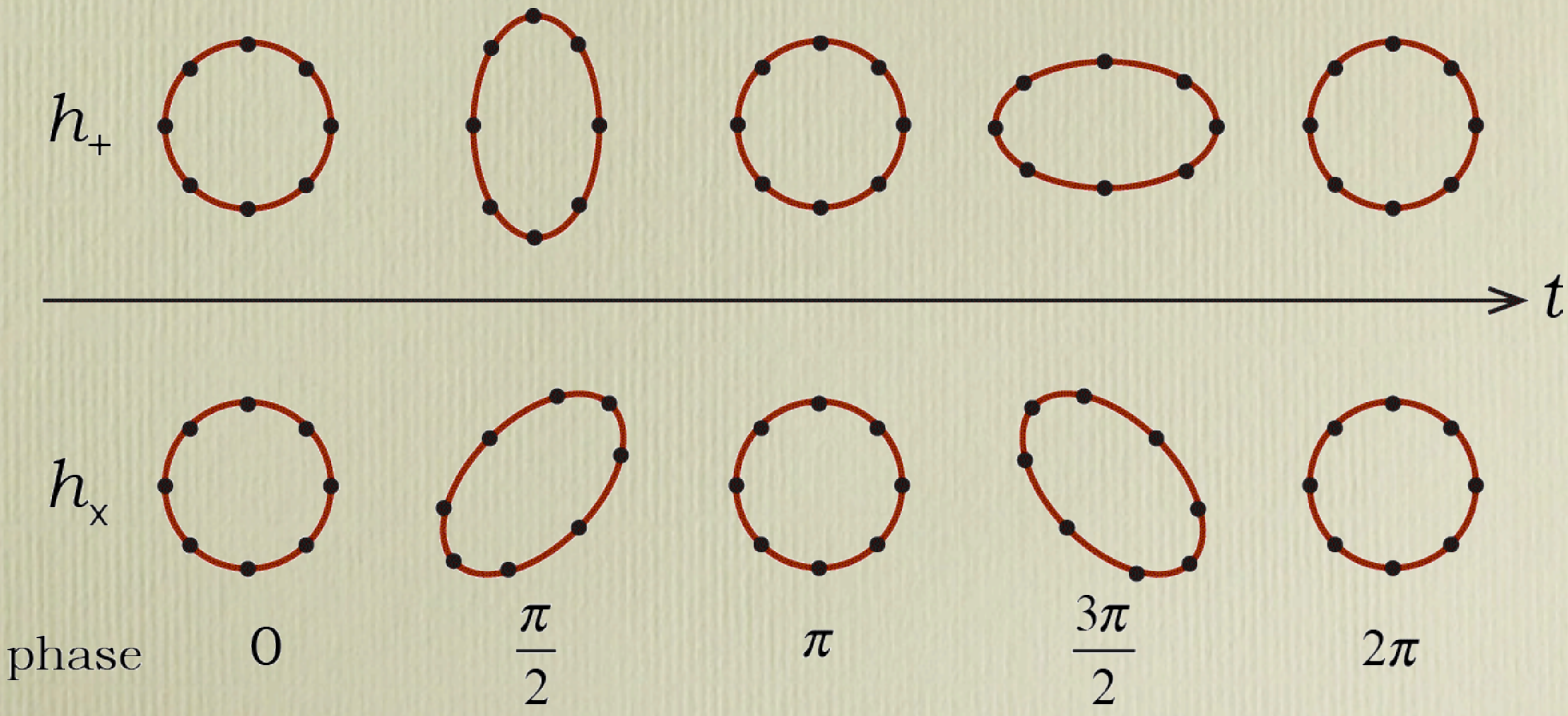
$$h_{xx}^{\mathrm{TT}} = -h_{yy}^{\mathrm{TT}} = h_{+}^{\mathrm{TT}} = A_{+}^{\mathrm{TT}} \cos\left(\omega(t - z/c)\right)$$
$$h_{xy}^{\mathrm{TT}} = h_{yx}^{\mathrm{TT}} = h_{\times}^{\mathrm{TT}} = A_{\times}^{\mathrm{TT}} \cos\left(\omega(t - z/c)\right)$$

- There are only two independent components;

h_+ and $h_\times$

Polarization

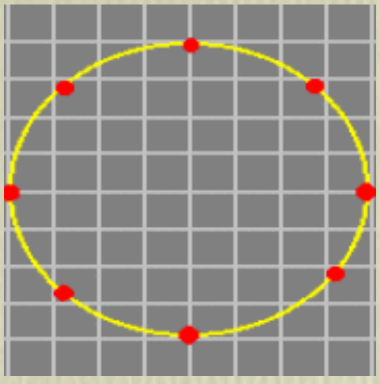
- GW of h_+ or $h_\times$ traveling in a direction perpendicular to the screen will cause particles to move like this:



Polarization

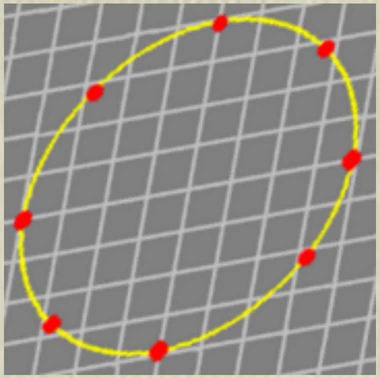
- **the + (plus) mode**

$$h_{+}^{\mathrm{TT}} \neq 0, \quad h_{\times}^{\mathrm{TT}} = 0$$



- **the $\times$ (cross) mode**

$$h_{+}^{\mathrm{TT}} = 0, \quad h_{\times}^{\mathrm{TT}} \neq 0$$



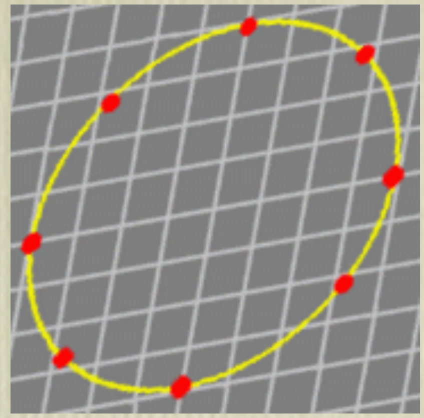
If only + mode or x mode wave comes,
each particle oscillates along a straight line.
The x mode is the same as + mode if rotated by 45 degrees.

These modes correspond to the linear polarization of light.

Polarization

- **the circular polarization**

$$h_{+}^{\mathrm{TT}} = h_{\times}^{\mathrm{TT}} \neq 0$$



In case of wave containing both + and x modes with the same amplitude, each particle moves along a circle.

It corresponds to the circular polarization.

- If their amplitude are not same, each particle moves along a elliptic. It corresponds to the elliptic polarization.

Properties of GWs

Like EM waves

- GWs are transverse.
- GWs propagate with the speed of light.
- Amplitude decrease as $\sim 1/r$.

While the lowest multipole of EM waves is the dipole

- The lowest allowed multipole of GWs is the quadrupole;
  - no monopole radiation
    as a result of mass conservation
  - no dipole radiation
    as a result of momentum conservation

Quadrupole Formula

- On the assumption that
  - (week field)
    the gravitational field generated by the source is sufficiently small
  - (low velocity)
    the typical velocities inside the source are small compared to the speed of light,

gravitational waves generated by the source are given by **the quadrupole formula**;

$$h_{ij}^{\mathrm{TT}}(t, \boldsymbol{x}) = \frac{1}{r} \frac{2G}{c^4} \ddot{Q}_{ij}^{\mathrm{TT}}(t - r / c)$$

Quadrupole Formula

- $h_{ij}^{\mathrm{TT}}(t,\boldsymbol{x}) = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}_{ij}^{\mathrm{TT}}(t-r/c)$

  Q_{ij}^{TT} : the TT part of the reduced quadrupole moment Q_{ij}

  $$Q^{ij} = \int d^3x\rho(t,\boldsymbol{x})\left(x^ix^j - \frac{1}{3}r^2\delta^{ij}\right) \quad ; \qquad Q_{ij}^{\mathrm{TT}} = \Lambda_{ij,kl}Q_{kl}$$

  $$\Lambda_{ij,kl} \equiv P_{ik}P_{jl} - \frac{1}{2}P_{ij}P_{kl} \ , \qquad P_{ij} = \delta_{ij} - n_in_j \ , \qquad n^i = \frac{x^i}{r}$$

- **The GW luminosity is given by**

  $$\frac{dE_{\mathrm{GW}}}{dt} = \frac{c^3r^2}{32\pi G}\int d\Omega\left\langle \dot{h}_{ij}^{\mathrm{TT}}\dot{h}_{ij}^{\mathrm{TT}}\right\rangle = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle$$

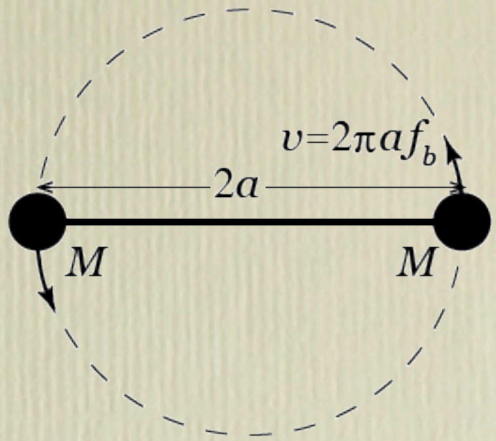
- **The angular momentum carried away by the GWs**

  $$\frac{dJ^i_{\mathrm{GW}}}{dt} = \frac{2G}{5c^5}\epsilon^{ikl}\left\langle \ddot{Q}_{ka}\dddot{Q}_{la}\right\rangle$$

Man-made sources

To examine how large or small the GWs are,

- Imagine creating **a wave generator**:
  - two masses of M each
    at opposite ends of a beam of $2a$
  - Rotate the beam at frequency f_b.



- **The luminosity of the GW:**

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij} \right\rangle = \frac{128G}{5c^5} M^2 a^4 \left(2\pi f_b\right)^6$$

$$= 5\times 10^{-24}\,\mathrm{erg/s} \left(\frac{M}{10^3\,\mathrm{kg}}\right)^2 \left(\frac{2a}{10\,\mathrm{m}}\right)^4 \left(\frac{f_b}{10\,\mathrm{Hz}}\right)^6$$

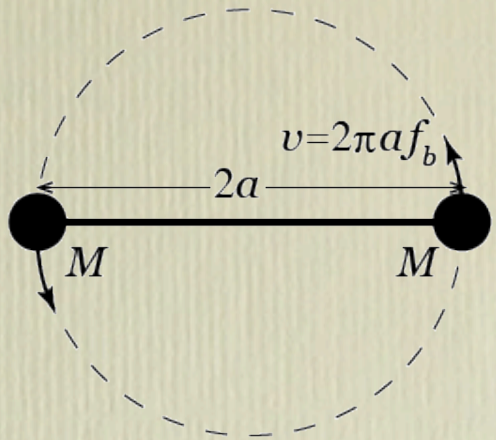
$$\left(5\times 10^{-31}\,\mathrm{W}\right)$$

It is difficult to realize whether it is small or large.

Man-made sources

- The amplitude of GW is given by

$$h_{ij}^{\mathrm{TT}} = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}_{ij}^{\mathrm{TT}} \sim \frac{8G}{c^4}\frac{Ma^2\left(2\pi f_b\right)^2}{r}$$

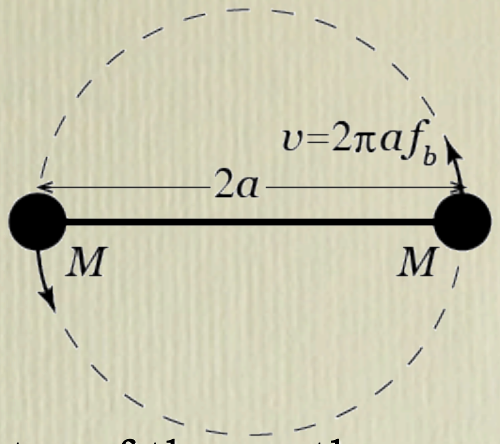


- To detect GWs, the detector must be at wave zone,

  that is, at least one wavelength from the source.
  Otherwise, the DC part or monopole part of the Newtonian gravity, which oscillates too due to circular motion of particles, dominates the waves.

Man-made sources

$$h_{ij}^{\rm TT} = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}_{ij}^{\rm TT} \sim \frac{8G}{c^4}\frac{Ma^2\left(2\pi f_b\right)^2}{r}$$



- If the beam rotates 10 times per second, the frequency of the GWs is 20Hz (twice of the freq. of particle rotation).
- The wavelength will be 15,000km ~ to the diameter of the earth.

- The amplitude at $r > \frac{c}{f_{\rm GW}} = \frac{c}{2f_b}$ becomes

$$h < \frac{64\pi^2 G}{c^5} M a^2 f_b^3 = 5\times 10^{-43}\left(\frac{M}{10^3\,{\rm kg}}\right)\left(\frac{2a}{10\,{\rm m}}\right)^2\left(\frac{f_b}{10\,{\rm Hz}}\right)^3$$

It is too small to detect.

Radiation from a Binary System

We must consider astronomical sources.
The most promising one is a binary star system.

- For a binary star system, replace $2M$ in the previous expression by the reduced mass μ.

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{32G}{5c^5}\mu^2 a^4 \left(2\pi f_b\right)^6 \qquad h = \frac{4G}{c^4}\frac{\mu a^2 \left(2\pi f_b\right)^2}{r}$$

- Using Kepler's law, $GM = (2\pi f_b)^2 a^3$, where M is the total mass of the system,

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{32G^{7/3}}{5c^5} M_c^{10/3} \left(2\pi f_b\right)^{10/3} \qquad h = \frac{4G^{5/3}}{c^4}\frac{M_c^{5/3}\left(2\pi f_b\right)^{2/3}}{r}$$

where we define the chirp mass M_c as

$$M_c \equiv \mu^{3/5} M^{2/5} = q^{3/5} M, \quad q = \mu / M$$

Radiation from a Binary System

- Since the GWs carry away the energy and the angular momentum from the system,
  the radius of the orbit decreases as

$$\frac{da}{dt} = -\frac{64G^3}{5c^5}\mu M^2 a^{-3} = -\frac{64G^3}{5c^5} M_c^{5/3} M^{4/3} a^{-3}$$

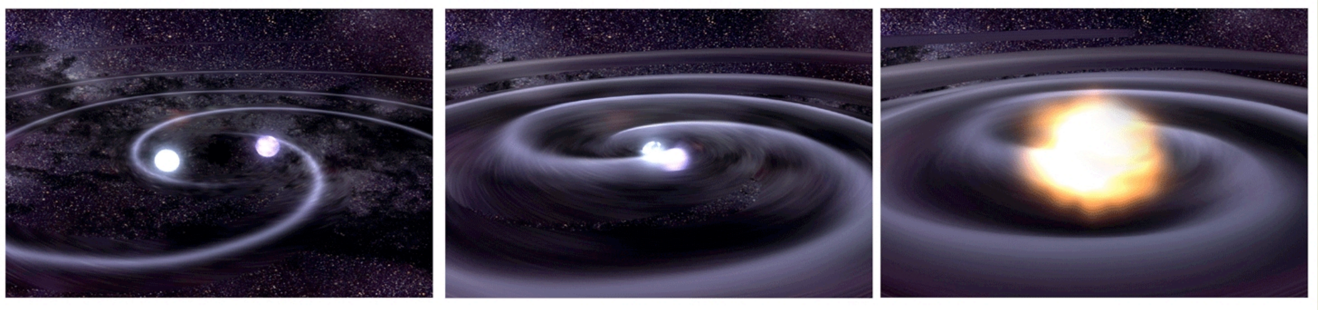
- The orbital frequency f_b increases and
  the orbital period $P_b = 1/f_b$ decreases as

$$\frac{df_b}{dt} = \frac{48G^{5/3}}{5\pi c^5} M_c^{5/3} \left(2\pi f_b\right)^{11/3}$$

$$\frac{dP_b}{dt} = -\frac{192G^{5/3}}{5c^5} M_c^{5/3} \left(\frac{P_b}{2\pi}\right)^{-5/3}$$

A Chirp Signal

- **a binary system consisting of neutron stars or black holes:**
  the radiation of energy by the orbital motion
  ➡ an inspiral orbit ➡ final merger of two stars

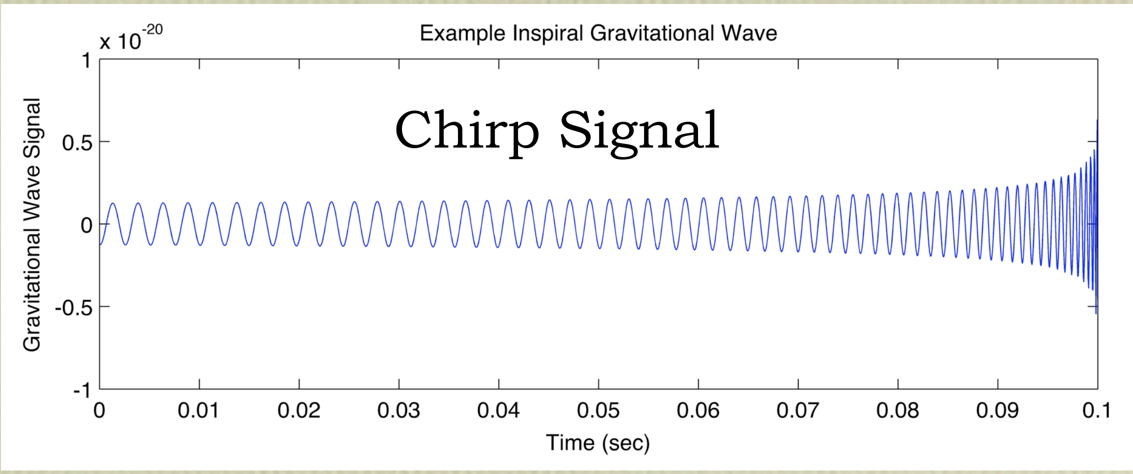


(http://www.ligo.org/science/GW-Inspiral.php)

For a binary system consisting of neutron stars or black holes, the radiation of energy by the orbital motion causes the orbit to shrink to make an inspiral orbit and finally two stars merge into one.

A Chirp Signal

- Frequency and amplitude of GWs: increasing



The shrinking will make GWs increase in freq. and amp.

This is called a chirp signal.

A Chirp Signal

- The rate of change in the orbital period is given by

$$\left|\frac{dP_b}{dt}\right| = 0.01\left(\frac{M_c}{1.2M_\odot}\right)^{5/3}\left(\frac{P_b}{0.01\,\mathrm{s}}\right)^{-5/3}$$

- A binary system consisting two NSs of mass $1.4M_\odot$ each ($M_c = 1.2M_\odot$)
- the time before coalescence τ_c ~ 0.3 sec when the orbital period ~ 0.01sec
- ***N_c* ~100**

  (Nc: the number of cycle for which the orbit changes significantly)

  ***N_c* ~10** even at τ_c ~ 0.01 sec

- the inspiral motion = **quasi-stationary** just before the circular orbit becomes unstable (the innermost stable circular orbit (ISCO))

Elliptic Orbits

- In an elliptic Keplerian orbit of eccentricity e,

$$\frac{dE_{\rm GW}}{dt} = \left(\frac{dE_{\rm GW}}{dt}\right)_{e=0} \times f(e); \qquad f(e) = \frac{1}{(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

- We can write the evolution of the semimajor axis a and the eccentricity e as

$$\frac{da}{dt} = \left(\frac{da}{dt}\right)_{e=0} \times f(e) = -\frac{64}{5}\frac{G^3\mu M^2}{c^5 a^3} f(e)$$

$$\frac{de}{dt} = -\frac{304}{15}\frac{G^3\mu M^2}{c^5 a^4}\frac{e}{(1-e^2)^{5/2}}\left(1 + \frac{121}{304}e^2\right)$$

Elliptic Orbits

- $$\frac{da}{de} = \frac{12a}{19}\frac{1+(73/24)e^2+(37/96)e^4}{e(1-e^2)[1+(121/304)e^2]}$$

  This equation can be integrated analytically, and gives

  $$a(e) = c_0 g(e), \qquad g(e) = \frac{e^{12/19}}{1-e^2}\left(1+\frac{121}{304}e^2\right)^{870/2299}$$

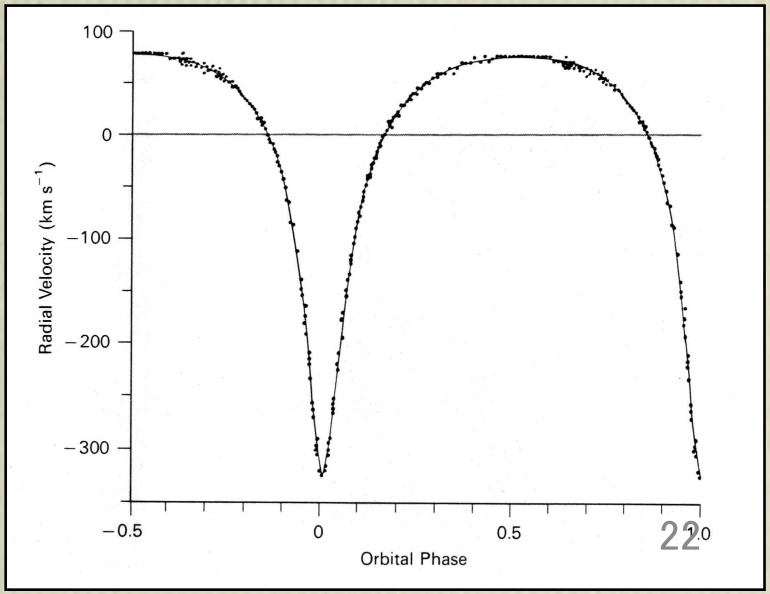
  c_0 ⬅ the initial condition; $a=a_0$ when $e=e_0$.

- If e is small while e_0 is not small, $e \approx \left[\frac{a}{a_0} g(e_0)\right]^{19/12}$.
- For the Hulse-Taylor binary pulsar (discussed later),
  $a_0 \simeq 2\times 10^9$ m, $e_0 \simeq 0.617$.
  When $a = O(10^2 R_{NS}) \simeq 10^3$ km, we have $e \simeq 6\times 10^{-6}$.

- the ellipticity has become zero
  = **the two stars move on a circular orbit**
  long before the two NSs approach the meger phase

PSR B1913+16 (the Hulse-Taylor Pulsar)

- In 1974, Russell Hulse and Joseph Taylor discovered a binary pulsar at Arecibo Observatory.
- the pulsar period P: 59 ms
- Doppler shift in observed period due to orbital motion.
  - Orbital period: P_b = 7h45m
  - Eccentricity: e = 0.617
- It is the first binary pulsar.



PSR B1913+16 (the Hulse-Taylor Pulsar)

- The effects of general relativity:
  - Periastron advance: 4.226598(5) deg/year
    - $M = m_p + m_c$

cf. Perihelion advance of Mercury
43 arcsec/century

  - Time dilation: 4.2992(8) ms
    (Gravitational redshift + Transverse Doppler)
    - $m_c(m_p + 2m_c)M^{-4/3}$
  - Orbital period decay: $-2.396(6) \times 10^{-12}$
    - $m_p m_c M^{-1/3}$

$$m_p = (1.4398 \pm 0.0002) M_\odot$$

$$m_c = (1.3886 \pm 0.0002) M_\odot$$

(Weisberg and Taylor 2010)

PSR B1913+16 (the Hulse-Taylor Pulsar)

- The rate is estimated from known orbital parameters and masses of the two stars using GR.

$$\dot{P}_b(\text{th}) = (2.402531 \pm 0.00001) \times 10^{-12}$$

- The observed value is

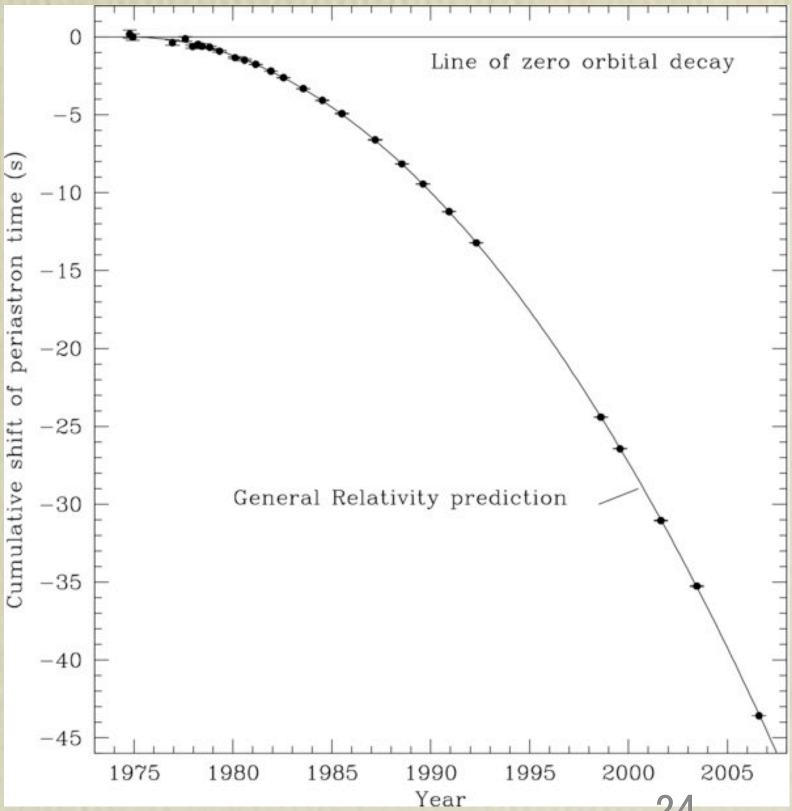
$$\dot{P}_b(\text{ob}) = (2.396 \pm 0.006) \times 10^{-12}$$

- The ratio is

$$\frac{\dot{P}_b(\text{ob})}{\dot{P}_b(\text{th})} = 0.997 \pm 0.002$$

➢ **Confirmation of GR.**

➢ **First observational evidence for GWs.**



PSR B1913+16 (the Hulse-Taylor Pulsar)

- The GWs by the inspiral of PSR B1913+16 (distance = 6,400pc)
  - amplitude: $h \sim 10^{-23}$
  - frequency: 7×10^{-5} Hz
  - wavelength: 4×10^{14} cm ~ 3,000 AU

- It is impossible to detect them at present by grand-base interferometer (LIGO, VIRGO, KAGRA) or space interferometer (LISA, DECIGO).

- Time to coalescence: ~ 250 Myr

PSR B1913+16 (the Hulse-Taylor Pulsar)

- First discovery of a **binary pulsar**
- First observational evidence for **GWs**
- First accurate determinations of **NS masses**
- Confirmation of **general relativity** as an accurate description of strong-field gravitational interaction

Nobel Prize for Taylor and Hulse in 1993

PSR J0737-3039

- Discovered in 2004.
- The first double pulsar;
  both stars are observed as pulsars
  PSR J0737-3039A and PSR J0737-3039B.
  (P = 22ms) (P = 2.7s)
  - Pulsar B disappeared in 2009 due to geodetic precession.
  - Come back in 2024.
- Orbital parameters:

  (PSR B1913+16)

  - orbital period: $P_b = 2.4$ h (7.75 h)
  - orbital velocity: $v_b = 0.001c$!!
  - eccentricity : $e = 0.088$ (0.617)

PSR J0737-3039

- Relativistic parameters

	Measured value	GR value	(PSR B1913+16)
* Orbital decay(ms) $\dot{P}_b$	$(-1.252 \pm 0.017) \times 10^{-12}$	-1.248×10^{-12}	(-2.4×10^{-12})
* Perast. adv. (deg/y) $\dot{\omega}$	16.8995 ± 0.0007	$-$	(4.2)
* Time dilation (ms) γ	0.386 ± 0.003	0.3842	(4.3)
* Shapiro delay (μs) r	6.2 ± 0.3	6.15	
* Shapiro delay $s = \sin \iota$	$0.99974\,^{+16}_{-39}$	0.99987	

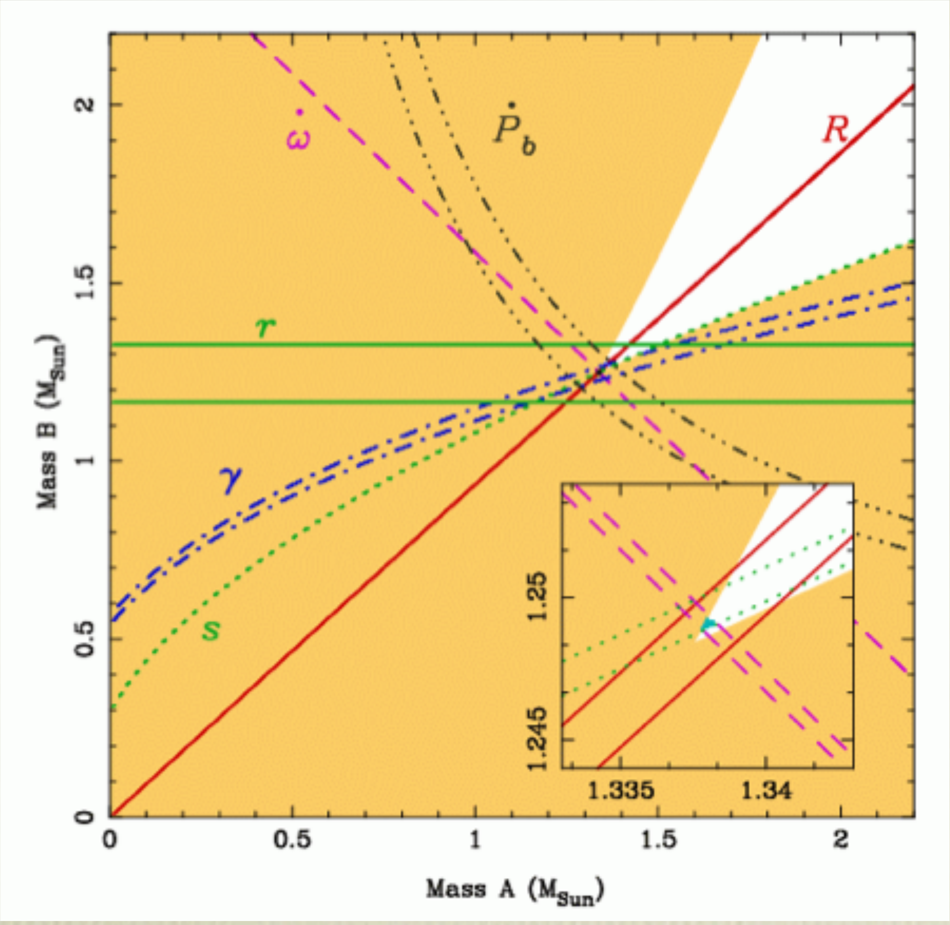
$M_A = 1.338 \pm 0.001 M_\odot$

$M_B = 1.249 \pm 0.001 M_\odot$

(Kramer et al. 2006)

PSR J0737-3039 Post-Keplerian Effects

- R: Mass ratio
- $\dot{\omega}$: periastron advance
- γ: gravitational redshift
- r & s: Shapiro delay
- $\dot{P}_b$: orbit decay



- Six measured parameters - only two independent
- Fully consistent with general relativity (0.1%)

Sources of Gravitational Waves

- Man made gravitational radiation: undetectable
- Compact & high-energy astronomical phenomena: for terrestrial detectors (observation band ~10Hz-1kHz)

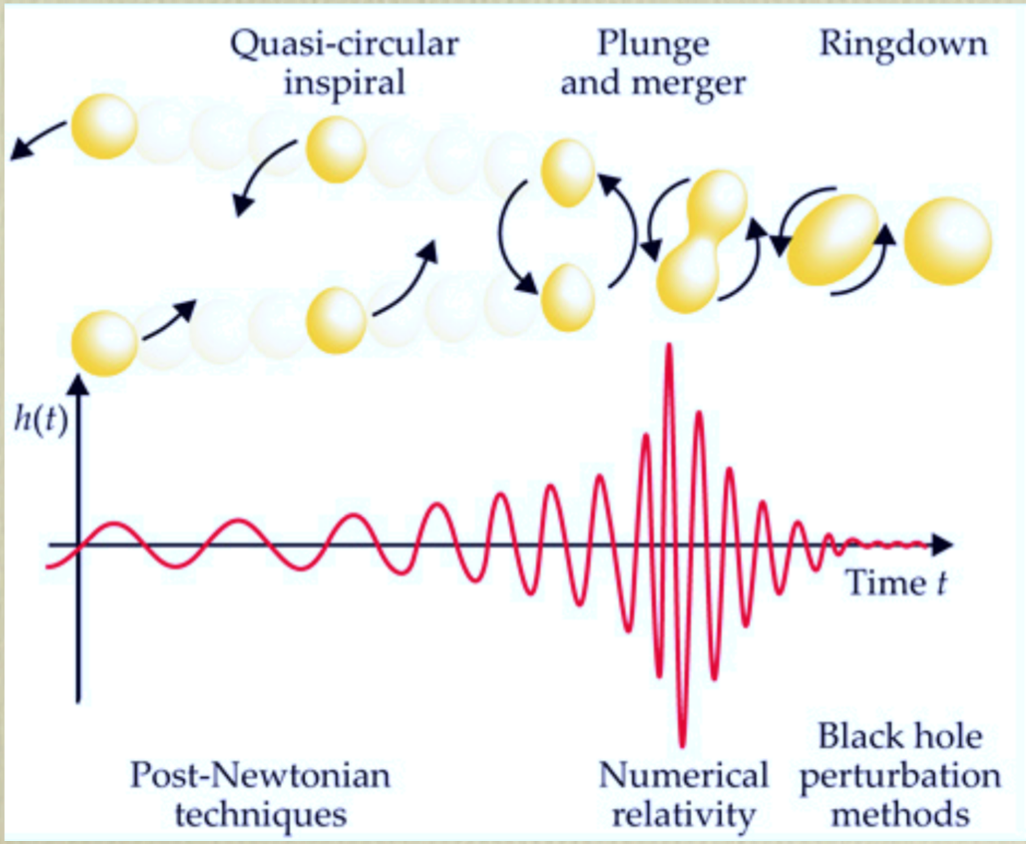
Compact Binary Coalescence

- Compact binary coalescence (CBC) is the most promising source of GWs for terrestrial and space interferometers.

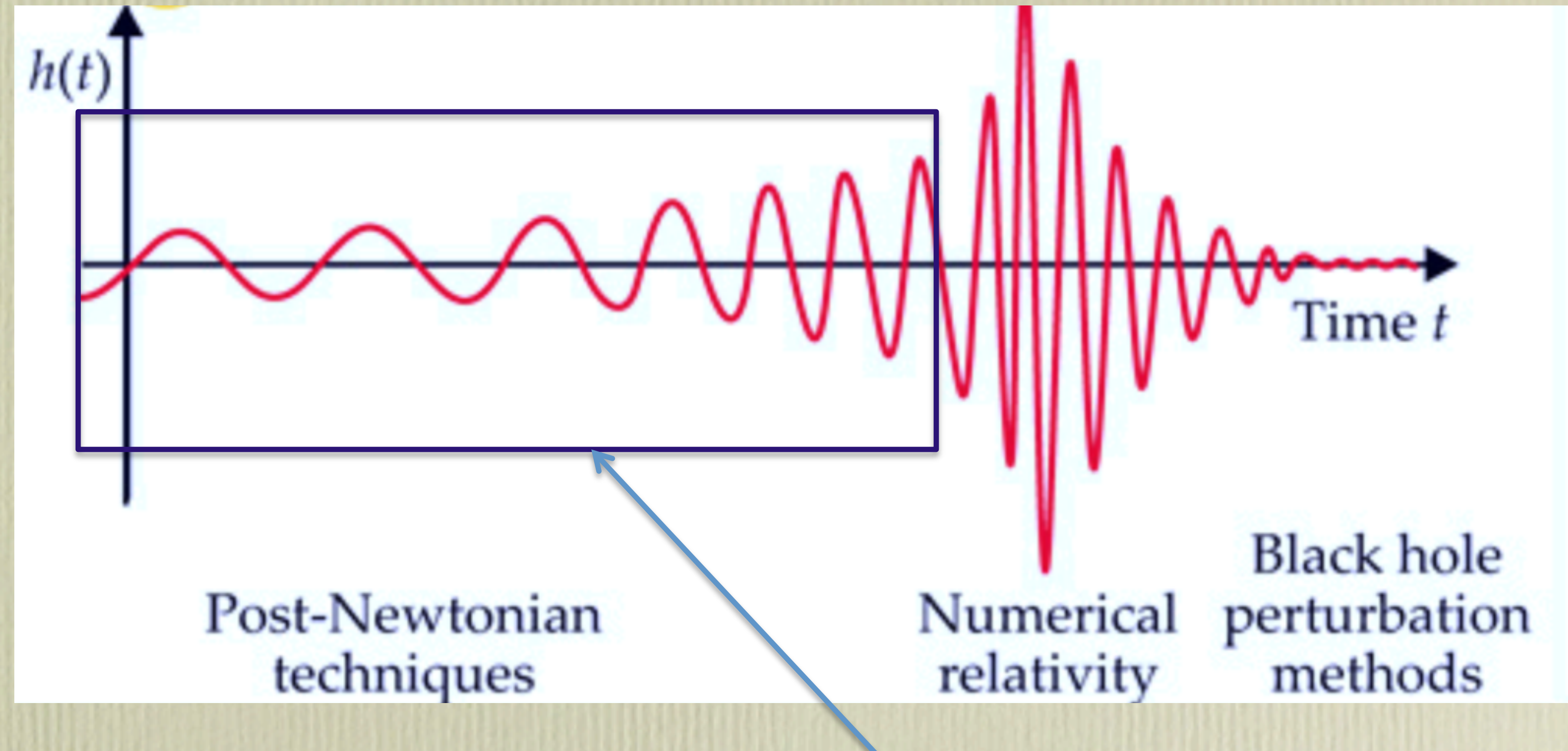
- Compact binary: a binary system consisting of neutrons stars and/or black holes.
  - NS/NS binaries
  - NS/BH binaries
  - BH/BH binaries

Compact Binary Coalescence

- **Three phase of waveform from CBC:**
  - a gradual inspiral
  - a rapid merger
  - ringdown of the resulting black hole or neutron stars

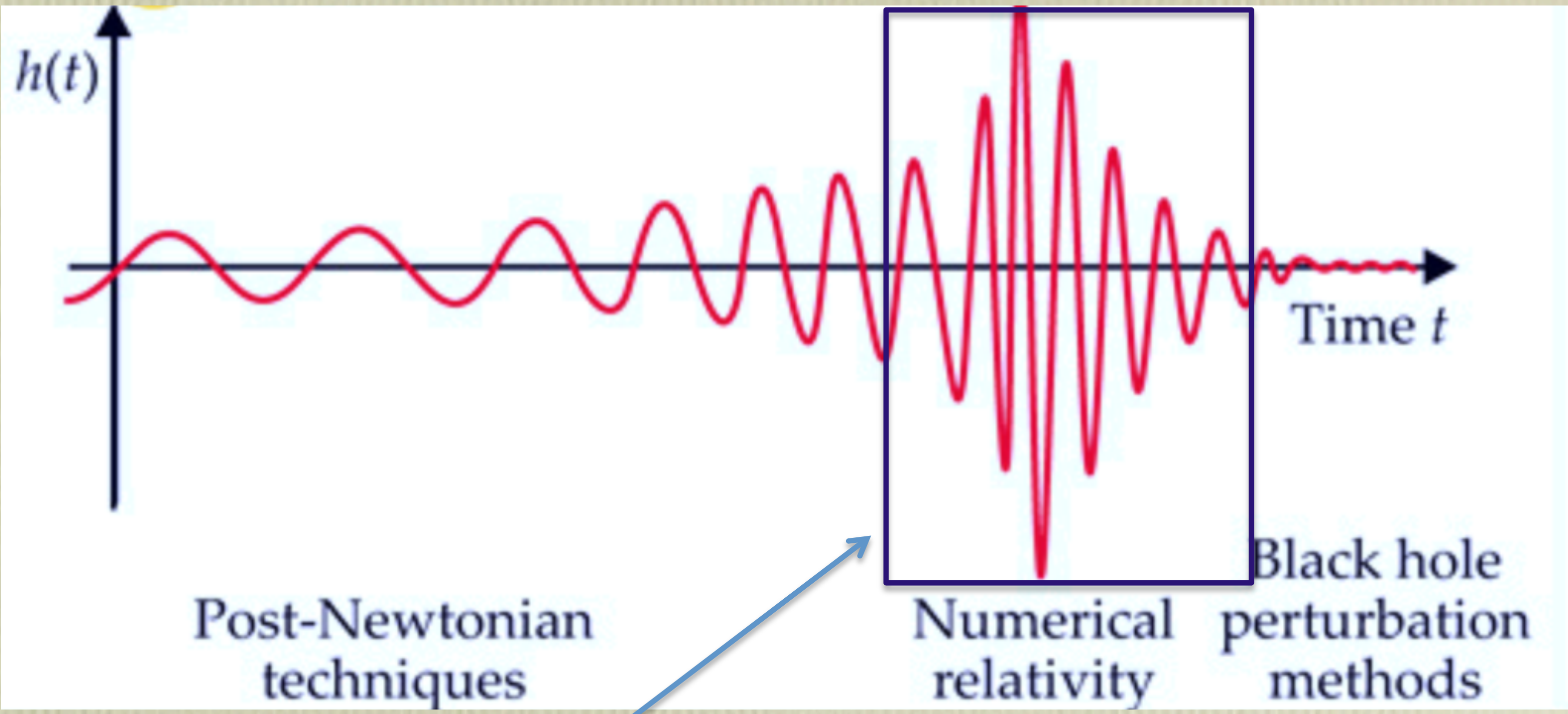


Inspiral Phase



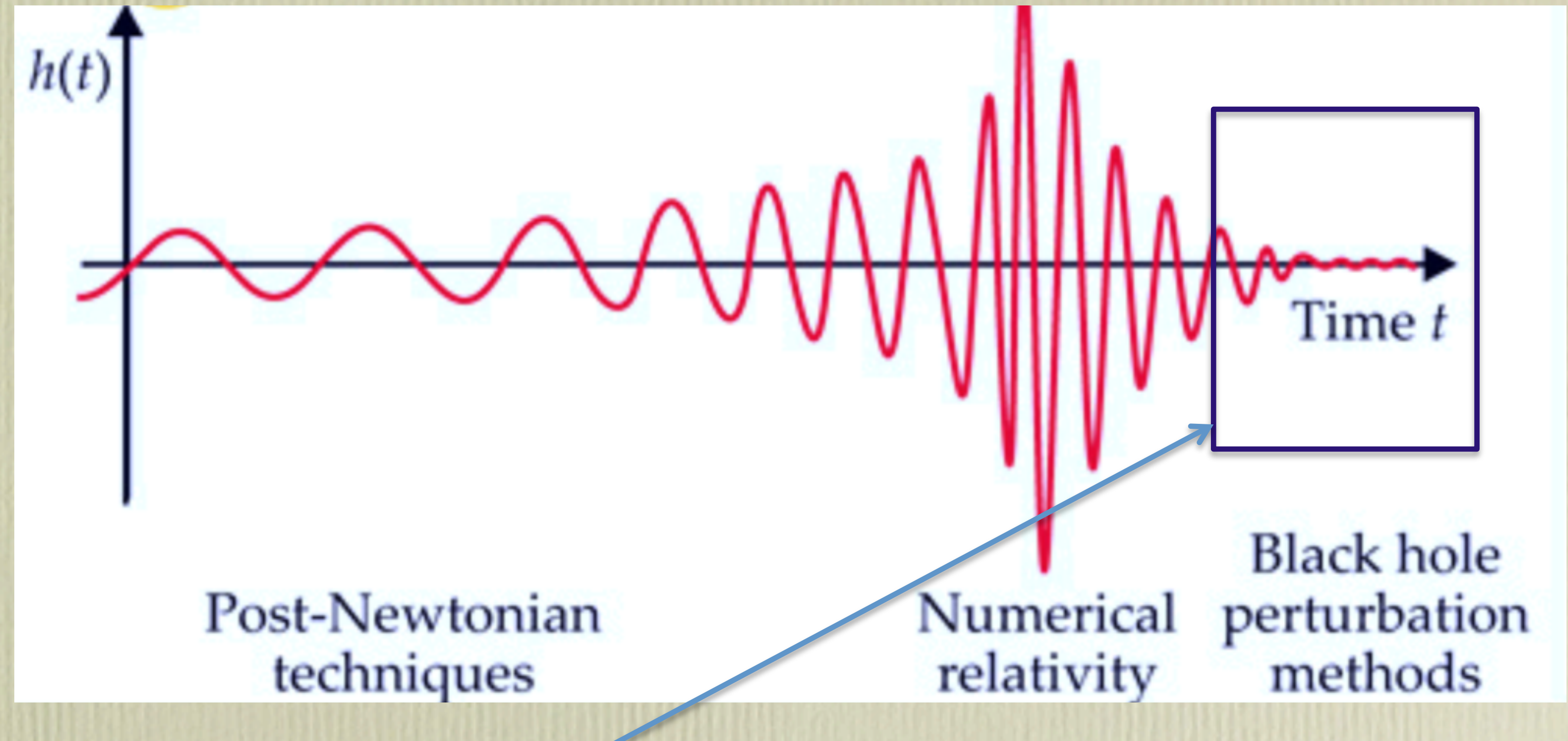
- GWs from the inspiral phase = **chirp signals**.
- Waveform (in the lowest order) ⟵ **the quadrupole formula**
- + **Higher-order post-Newtonian effects** and **contributions of higher multipole** (Tagoshi's lecture)

Merger Phase



- the merger phase: **numerical relativity**
- Various physics from GWs in this phase
  - Equation of State (EOS) of neutron star matter
  - Electromagnetic radiation, which may lead gamma-ray burst
  - Neutrino emission (1st half of the school)

Ringdown Phase



- **Dumped oscillation** in ringdown phase
- **The quasi-normal modes** of a resultant BH
- the frequency and dumping rate ⟵ **only on mass and spin** (**the perturbation calculation** of BH spacetime)
- GWs in ringdown phase ⟶ the properties of BH

GWs from CBC

- Frequency bands of GWs from CBC are
  - NSs or stellar mass BHs:
    f = 10 ~ 10^3 Hz (λ = 300 ~ 30,000 km)
  - supermassive BHs
    f = 10^{-4} ~ 10^{-1} Hz (λ = 3×10^6 ~ 3×10^9 km)
    0.02 ~ 20AU

- You can listen to the "sound" of GWs from coalescence of stellar mass compact binaries,
  if a speaker is connected to the output of the detector.
  (cf. audio frequency: 20 ~ 20,000 Hz)

Merger Rates

- The merger rate of NS/NS binaries:
  from Galactic compact NS/NS binaries observed
- 5 binaries in the field
  1 binary in a globular cluster

PSR	t_{mrg}/Gyr	$M_1/M_\odot$	$M_2/M_\odot$	
J0737-3039	0.09	1.34	1.25	field (double PSR)
B2127+11C	0.22	1.36	1.38	cluster
J1906+0746	0.30	1.25	1.37	field
B1913+16	0.33	1.44	1.39	field
J1756-2251	1.7	1.39	1.18	field
B1534+12	2.7	1.33	1.35	field

- Galactic merger rate estimated: $3\sim190\times10^{-6}$ yr^{-1}.
- It implies AdvLIGO event rates in the range $7\sim400$ yr^{-1}.

(Kim et al. 2010)

Merger Rates

- BH/BH or BH/NS binaries have not observed.
- The merger rates of them (including NS/NS binaries) are estimated based on models of compact binary evolution.

Model	NS-NS	BH-NS	BH-BH	comments
S	3.9 (1.3)	9.7 (5.1)	7993.4 (518.7)	standard
V5	3.9 (1.3)	9.4 (4.8)	8057.8 (533.7)	$M_{\rm NS,max} = 3\ {\rm M}_\odot$
V6	3.9 (1.3)	9.3 (4.7)	8041.7 (523.6)	$M_{\rm NS,max} = 2\ {\rm M}_\odot$
V7	5.0 (1.5)	14.8 (8.3)	8130.1 (574.2)	half NS kicks
V8	3.9 (1.3)	1.2 (0.3)	172.2 (14.0)	high BH kicks
V9	3.9 (1.3)	11.8 (6.7)	8363.6 (654.9)	no BH kicks
V10	5.2 (1.7)	5.7 (4.9)	7762.7 (487.0)	delayed SN
V11	3.9 (1.1)	10.5 (6.3)	12434.4 (888.1)	low winds
V12	11.7 (0.8)	7.6 (5.8)	8754.6 (275.3)	RLOF: conservative
V13	3.7 (0.9)	76.9 (62.1)	1709.6 (966.1)	RLOF: non-conservative

(Belczynski 2013)

Merger Rates

- Detection rates for CBC with AdvLIGO

	N_{low} yr^{-1}	N_{re} yr^{-1}	N_{high} yr^{-1}
NS-NS	0.4	40	400
NS-BH	0.2	10	300
BH-BH	0.4	20	1000

(Adadie et al. 2010)

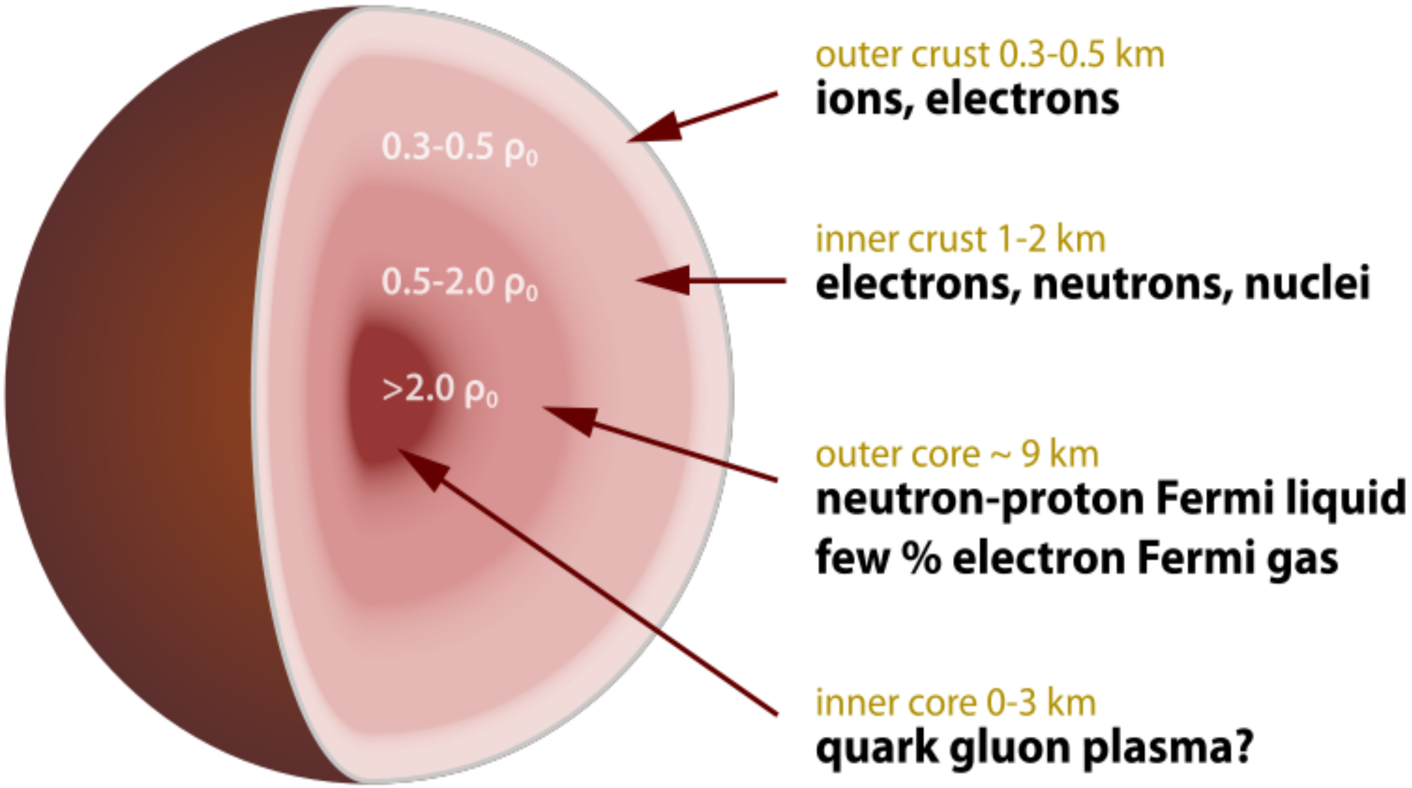
the lower end, the most realistic and the upper end of estimated detection rate with optimal horizon distance 445Mpc for NS-NS, 927Mpc for NS-BH and 2187Mpc for BH-BH

(Astro)physics from CBC

- Detection of GWs will open up a new world in physics and astrophysics.
- The most important is test of dynamical gravity; GR of Einstein or alternative theories?
  - the rates of energy and angular momentum release
  - waveforms
  - dipole radiation ...... etc.
- We shall discuss some topics of physics and astrophysics obtained with GWs from CBC.

Equation of State of NS

- Neutron star composition is still unknown.

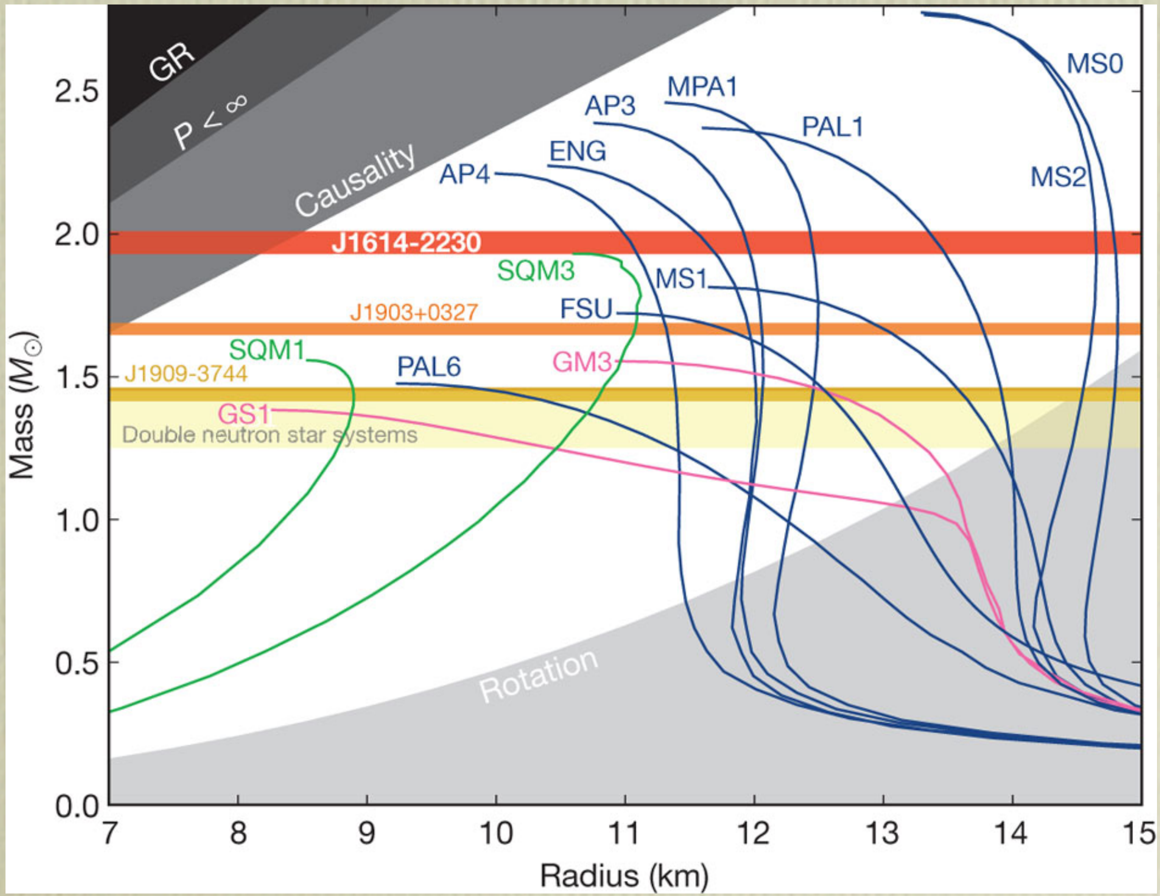


$\rho_0 = 2.8 \times 10^{14}$ g/cm^3

(Wikipedia)

Equation of State of NS

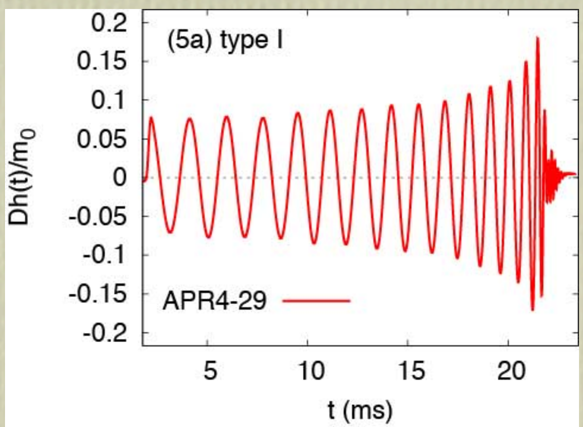
- EOS in super-nuclear density: known
  - many EOS of a neutron star
- The maximum mass of a neutron star ⬅ EOS
- the mass-radius relation, etc.



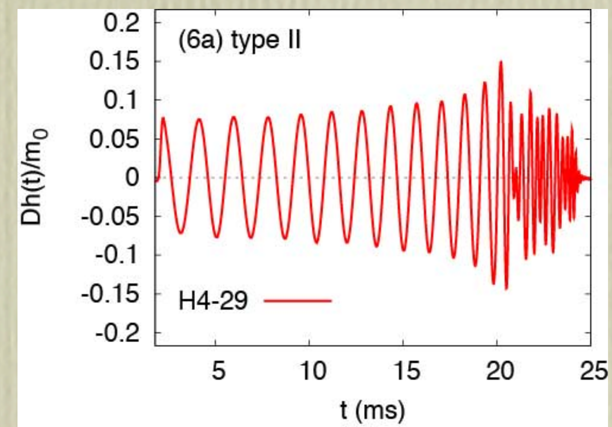
Demorest et al 2010

Equation of State of NS

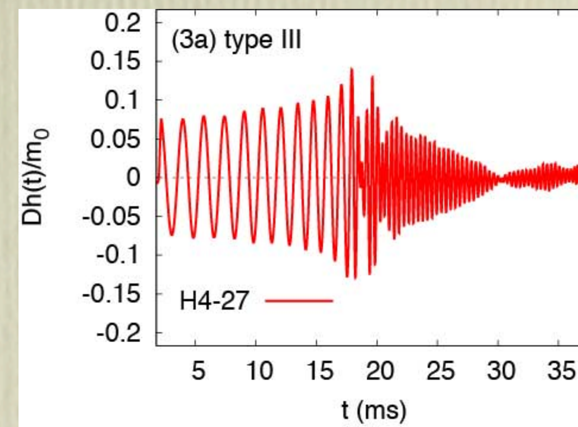
- The characteristics of the GWs from coalescence of a NS/NS binary depends on EOS of the NS.



A BH is formed directly; no HMNS.



A HMNS is formed temporally and it collapses to a BH.



A long-lived HMNS is formed and finally it collapses to a BH.

(Hotokezaka et.al. 2011)

We can place constraints on EOS of a NS.

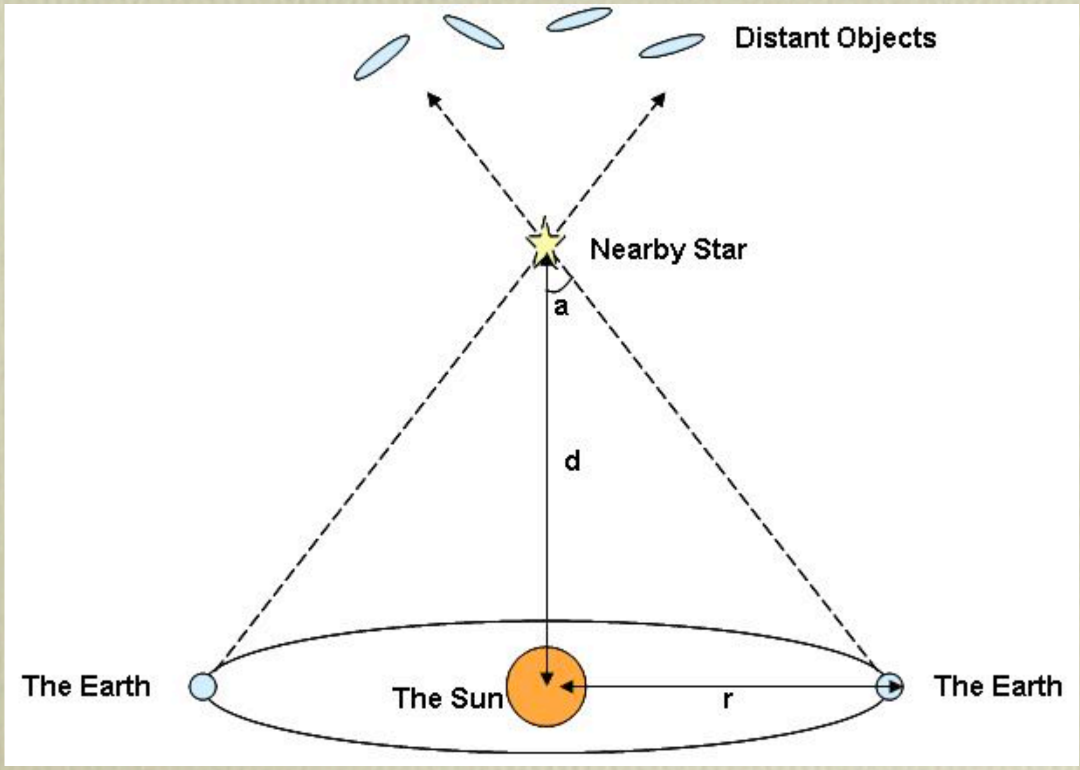
Distance Ladder

- **Determination of the distance** to astronomical objects
  ➡ properties of the objects
  & the universe including the Hubble constant

- **Distance ladder**:
  the succession of method
  for determining the distances to astronomical objects

Trigonometric Parallax

- **Trigonometric Parallax**
  (< 100pc ~1kpc: Hipparcos satellite)
  - motion of the earth around the sun
    apparent motion (parallax)
    of nearby stars against more distant objects



Standard Candles

- **Standard Candles:**
  objects for which the absolute magnitude is known

- Comparison of the apparent brightness
  with the absolute magnitude of objects
  ⟹ luminosity distances.

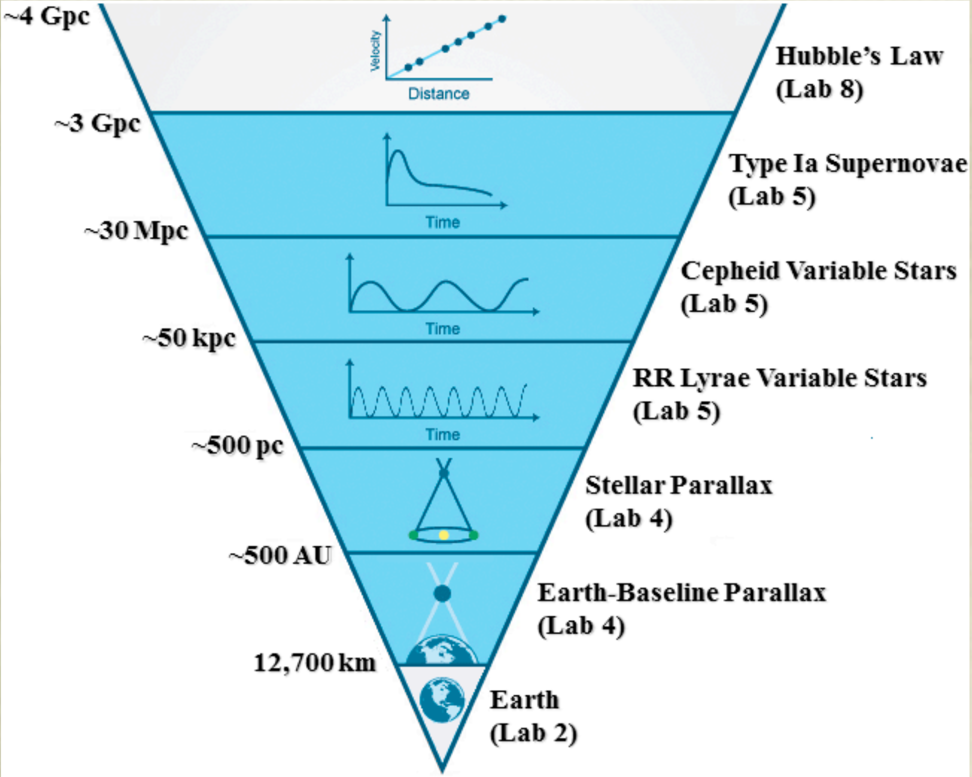
  - The standard candle at each step is calibrated by the candle at the previous step.

Standard Candles

★ Period-luminosity relations of variable stars
- RR Lyare (in GC)
- Cepheid
  (up to nearby galaxies)

★ Type Ia SNe

★ Tully-Fisher relation



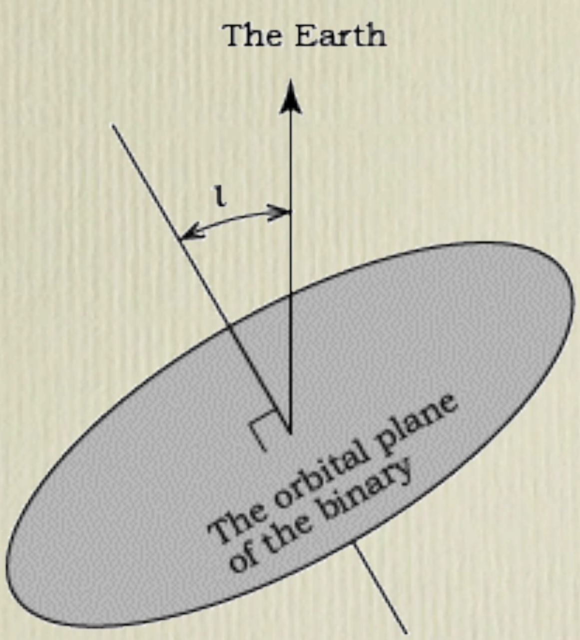
- The errors and the uncertainties increase at each steps of the distance ladder.

Chirp Signals as Standard Sirens

- B. Schutz (1986): measuring the distance of the source using chirp signal from CBC
- **standard sirens**: by analogy with the standard candles of electromagnetic astronomy
- GW from CBC:

$$h_+(t) = \frac{A}{r}\left(\frac{\pi f_{\rm gw}(\tau)}{c}\right)^{2/3} \frac{1+\cos^2\iota}{2}\cos\Phi(\tau)$$

$$h_\times(t) = \frac{A}{r}\left(\frac{\pi f_{\rm gw}(\tau)}{c}\right)^{2/3} \cos\iota\sin\Phi(\tau)$$



including the effect of the inclination ι , the angle between the line of sight and the direction normal to the orbit

Chirp Signal

$$h_+(t) = \frac{A}{r}\left(\frac{\pi f_{\mathrm{gw}}(\tau)}{c}\right)^{2/3} \frac{1+\cos^2\iota}{2}\cos\Phi(\tau)$$

$$h_\times(t) = \frac{A}{r}\left(\frac{\pi f_{\mathrm{gw}}(\tau)}{c}\right)^{2/3} \cos\iota\sin\Phi(\tau)$$

where

$$A = 4\left(\frac{GM_c}{c^2}\right)^{5/3}$$

$$f_{\mathrm{gw}}(\tau) = \frac{1}{\pi}\left(\frac{5}{256\tau}\right)^{3/8}\left(\frac{GM_c}{c^2}\right)^{-5/8}$$

$$\Phi(\tau) = \int_\tau^{\tau_0} 2\pi f_{\mathrm{gw}}(\tau)d\tau = -2\left(\frac{5GM_c}{c^2}\right)^{-5/8}\tau^{5/8} + \Phi_0$$

$\tau = t_{\mathrm{coal}} - t;\quad t_{\mathrm{coal}}$: time at coalescence

Standard Siren

$$\dot{f}_{gw} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f_{gw}^{11/3}$$

- **The chirp mass M_c** ⬅ the measured values of f_{GW} and $\dot{f}_{GW}$

$$h_+(t) = \frac{A}{r}\left(\frac{\pi f_{gw}(\tau)}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\Phi(\tau), \quad h_\times(t) = \frac{A}{r}\left(\frac{\pi f_{gw}(\tau)}{c}\right)^{2/3}\cos\iota\sin\Phi(\tau)$$

- h_+ and $h_\times$ ➡ **the distance r** to the source
  the inclination ι will be obtained.

- For a binary at a cosmological distance of redshift z,
  $M_c \longrightarrow (1+z)\, M_c$; $f_{gw} \longrightarrow f_{gw}/(1+z)$

the distance r $\longrightarrow$ **the luminosity distance $D_L(z)$**.

Standard Siren

- **the mass-inclination degeneracy problem:**
  (electromagnetic observation of binary star system)
  - the orbital period
  - the maximum radial velocity

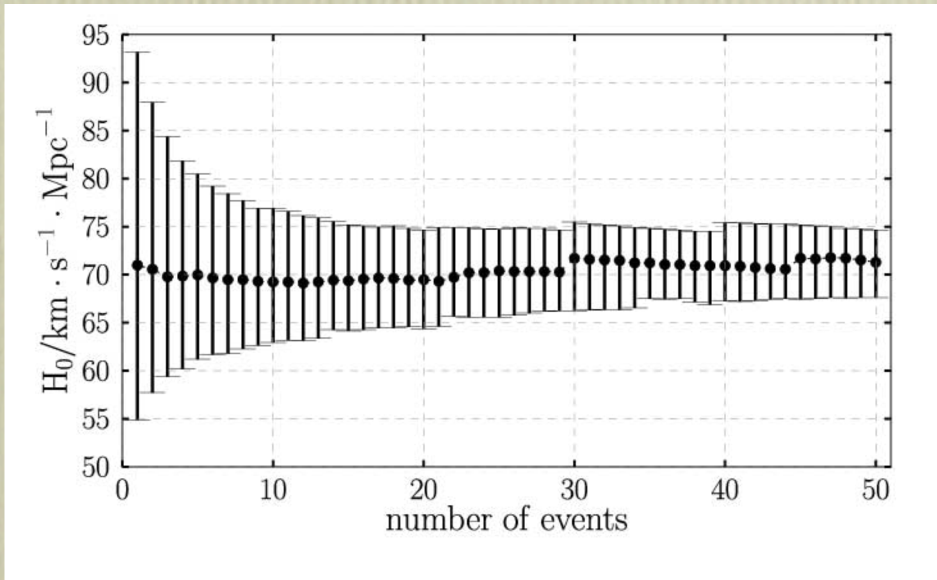
  + Kepler's law

  ➡ only $m \sin \iota$ is determined

- The measurement of ι is not easy in general.
- Observation of the GWs can determine the inclination independently of electromagnetic observation.

Measurement of H_0

- GW observation ➡ **the luminosity distance** independently of the distance ladder
- Uniquely clean and powerful way to measure the Hubble constant H_0
- Need redshift of the source
- Short gamma-ray bursts = binary NS-NS mergers:
  - ➢ determine H_0 within ~3% (D. Holz 2012)



Other Sources of GWs

- **GW bursts from gravitational collapse**
  - supernovae, (long) GRBs
  - information on explosion mechanism
    (connecting with EM and neutrino observations)
- **GWs from pulsars**
  - deformation from an axially symmetric shape
    ⇒ continuous wave
  - pulsar glitches ⇒ GW bursts (connecting with EM obs.)
- **Stochastic background**
  - From fundamental processes such as the Big Bang
  - Superposition of GWs from countless discrete system

  ➡ information on cosmology,
  especially on the very early universe

Multi-Messenger Observations

- **observation of counterparts**
  with electromagnetic waves and neutrino
  ➡ fruitful results on physics and astrophysics
  from GW observation
  multi-messenger observations

- developments in
  astrophysics through multi-messenger observation of gravitational wave sources in Japan (2012)
  - Search of X-ray and gamma-ray radiation from GW objects
  - Search of optical and IR phenomena in GW sources
  - Neutrino search from GW sources
  - Data analysis of GW searches
  - Theoretical research on GW objects

And ...

- In order to succeed the gravitational wave physics and astrophysics, we need
  - development in gravitational wave detectors (Kuroda's lecture)
  - data analysis strategy (lectures by Tagoshi, Hayama and KO)